

Upper limit to Ω_B in scalar-tensor gravity theories

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ABSTRACT

In a previous paper (Serna & Alimi 1996b), we have pointed out the existence of some particular scalar-tensor gravity theories able to relax the nucleosynthesis constraint on the cosmic baryonic density. In this paper, we present an exhaustive study of primordial nucleosynthesis in the framework of such theories taking into account the currently adopted observational constraints. We show that a wide class of them allows for a baryonic density very close to that needed for the universe closure. This class of theories converges soon enough towards General Relativity and, hence, is compatible with all solar-system and binary pulsar gravitational tests. In other words, we show that primordial nucleosynthesis does not always impose a very stringent bound on the baryon contribution to the density parameter.

Subject headings: cosmology:early universe, cosmology:dark matter, cosmology: theory

1. INTRODUCTION

One of the most convincing pieces supporting the standard hot big bang model for the early Universe is the excellent agreement between the predicted and observed primordial light element abundances. However, this agreement requires that the present baryon density ρ_b must be smaller (Walker et al. 1991, Copi, Schramm & Turner 1995a-1995b, Olive 1996), than about 10% of that needed for the Universe closure ($\Omega_B \equiv \rho_b/\rho_{crit} < 0.1$). A high-density universe ($\Omega \gg 0.1$), seems then to be impossible unless that most dark matter in the Universe were non-baryonic.

The problem in admitting large ρ_b values is that, in the standard primordial nucleosynthesis model, they lead to the overproduction of ^4He and ^7Li and the underproduction of deuterium. In order to understand this difficulty, we recall that primordial nucleosynthesis starts, in the early Universe, soon after that the cosmic temperature becomes smaller than that needed to maintain the proton-to-neutron ratio in its equilibrium value (freezing-out temperature). The nuclear reactions which then take place lead first to the D and ^3He formation. These elements are then burnt to produce ^4He . At the end of this process, essentially all neutrons are incorporated into ^4He , while just a small fraction of them remains in D and ^3He . Consequently, the ^4He abundance mainly depends on the fraction of neutrons existing at the beginning of the nucleosynthesis process or, equivalently, on the freezing-out temperature for the n/p ratio. On the other hand, since the burning of D and ^3He grows with the baryon density, the final yields of these two elements decrease with ρ_b , while those of ^4He and ^7Li increase¹.

Several attempts to relax the nucleosynthesis bound on Ω_B have been previously considered (Malaney & Mathews 1993). This is the case, for example, of inhomogeneous big bang models, and the attempts of modifying the nuclear reaction rates by the introduction of new decaying particles (e.g. Schramm 1991, Gyuk & Turner 1994). However, all these attempts have been unsuccessful or seem to be mere modifications to try to solve this problem. The nucleosynthesis bound on Ω_B is today considered as a unavoidable constraint which must be imposed even to study the formation of large-scale structures in the Universe.

Another possibility also analyzed in the literature consists of modifying the gravity description and, hence, the Universe expansion rate without altering the nuclear physics, the Universe composition or the conservation laws. This last possibility can be considered as more natural from a theoretical point of view

¹Although the lithium-7 history is much more complicated, when the baryon density is relatively large, its final abundance also increases with ρ_b due to the ^7Li production through ^7Be

because it would allow us to construct a global framework which is coherent with that often assumed to describe the epochs prior to primordial nucleosynthesis. As a matter of fact, to study the earliest times in the Universe evolution, the existence of an extra scalar field, in addition to the Einstein metric tensor, is usually assumed. Such gravity theories are termed scalar-tensor theories (Bergmann 1968, Wagoner 1970, Nordtvedt 1970), and are the simplest generalization of the Einstein theory. They provide a natural (non-fine-tuned) way to restore the original ideas of inflation while avoiding the cosmological difficulties coming from the vacuum-dominated exponential expansion obtained in General Relativity (GR) (La & Steinhardt 1989, Weinberg 1989, Steinhardt & Accetta 1990, Barrow & Maeda 1990, Liddle & Wands 1992, Deruelle, Gundlach & Langlois 1992, Garcia-Bellido & Wands 1992, Barrow 1995). Scalar-tensor theories also arise in the current theoretical attempts at deepening the connection between gravitation and the other interactions. For example, in modern revivals of the Kaluza-Klein theory and in supersymmetric theories with extra dimensions, one or more scalar fields arise in the compactification of these extra dimensions (Jordan 1949, Kolb, Perry & Walker 1986, Cho 1992, Wesson & Ponce de Leon 1995). Furthermore, scalar-tensor theories may also appear as a low-energy limit of superstring theories (Green, Schwarz & Witten 1988).

The Big Bang nucleosynthesis (BBN) in presence of a scalar field has been previously analyzed in some cases (Bekenstein & Meiseils 1980, Arai, Hashimoto & Fukui 1987, Serna, Dominguez-Tenreiro & Yepes 1992, Serna, Dominguez-Tenreiro 1993, Serna & Alimi 1996b). Among all these analyses, only some models pointed out by Serna & Alimi (1996b) were able to relax the nucleosynthesis bound on Ω_B . The key of such a result was that the expansion rate resulting in such theories avoided simultaneously the overproduction of ^4He and ^7Li and the underproduction of D. In this paper we construct a wide class of scalar-tensor gravity theories also allowing a large range for baryonic density.

2. SCALAR-TENSOR NUCLEOSYNTHESIS

Scalar-tensor theories are characterized by an arbitrary coupling function $\omega(\Phi)$, which determines the relative importance of the additional scalar field Φ . GR is the asymptotic case in which the coupling function is infinite and $\Phi = \text{constant} = 1$. Since GR reproduces very accurately the solar-system and binary pulsar dynamics (Will 1993), the behavior of any viable scalar-tensor theory must be restricted to be at present extremely close to that implied by GR. The physical conditions in the early Universe and

the subsequent cosmological evolution in the framework of these theories can be nevertheless very different from the usual ones.

In order to analyze the possible existence of scalar-tensor theories, able to reproduce the right primordial abundances for an Ω_B interval much wider than in GR, we must first calculate the resulting cosmological models. To that end, we consider an **isotropic and homogeneous** universe and a specific form for the coupling function. The line element has then a Robertson-Walker form and the energy-momentum tensor corresponds to that of a perfect fluid. In that concerning the coupling function, a convenient form is given by

$$|3 + 2\omega| = (3/\lambda^2)(x^{-1} + k) \quad (1)$$

where λ and k are arbitrary constants and $x = |\Phi - 1|$. As a matter of fact, as shown by Serna & Alimi (1996a), such a form gives an exact representation for most of the particular scalar-tensor theories proposed in the literature and, in addition, it contains all the possible early behaviors of any theory where $\omega(\Phi)$ is a monotonic, but arbitrary, function of Φ .

When such an $\omega(\Phi)$ function is introduced into the field equations, one essentially obtains four different classes of scalar-tensor theories. The two first classes correspond to singular models with a monotonic time evolution of the speed-up factor ($\xi \equiv H/H^{FRW}$, where H is the Hubble parameter, while H^{FRW} is that predicted by GR at the same temperature). In the first class, the Universe expansion is always faster than in GR ($\xi > 1$), while the second class is characterized by an expansion rate slower than in GR ($\xi < 1$). Nonsingular models or models with a critical temperature where $3 + 2\omega = 0$ constitute the third class of theories. Finally, the fourth class corresponds to models with a non-monotonic $\xi(T)$ function. These last theories have an initial phase where the Universe expansion is slower than in GR but, afterwards, it becomes faster than in the standard cosmology. Obviously, the Universe evolution in the framework of any viable scalar-tensor theory must satisfy $\xi \rightarrow 1$ at present.

In the first three classes of scalar-tensor theories, the right primordial yields are only obtained (Serna & Alimi 1996b) in the limit where such theories are almost indistinguishable from GR, from the beginning of primordial nucleosynthesis up to the present. Consequently, the allowed Ω_B interval is essentially the same as that implied by GR.

We will then focus on the primordial production of light elements in the framework of theories with a non-monotonic evolution of the speed-up factor (class-4 models). This class of models is inevitably obtained when $\lambda/\sqrt{k} > 1$, and both $3 + 2\omega$ and $(\Phi - 1)$ have positive values. We show below that new constraints on

the baryonic density parameter can then be obtained.

The computation of the primordial production of light elements² has been performed by using the updated reaction rates of Caughlan and Fowler (1988) and Smith *et al* (1993). We have considered $N_\nu = 3$, $T_0 = 2.73\text{K}$, $\tau_n = 889\text{s}$, $H_0 = 50\text{km s}^{-1}\text{Mpc}^{-1}$. Other runs have been also performed, with $\tau_n = 889.8 \pm 3.6\text{s}$ and $H_0 = 100\text{km s}^{-1}\text{Mpc}^{-1}$, in order to test the robustness of our conclusions. For each theory defined by the parameters λ and k , we have searched the largest Ω_B values for which there exists a present value of the coupling function ω_0 (i) compatible with all solar system experiments and (ii) leading to the observed primordial abundances (Copi, Schramm & Turner 1995b):

$$\begin{aligned} 0.221 &\leq Y_P \leq 0.243 \\ \text{D/H} &\geq 1.5 \cdot 10^{-5} \\ (\text{D} + {}^3\text{He})/\text{H} &\leq 1.1 \cdot 10^{-4} \\ 0.7 \cdot 10^{-10} &\leq {}^7\text{Li/H} \leq 3.5 \cdot 10^{-10} \end{aligned}$$

Some of these largest upper bounds on Ω_B are shown as isocontour lines on figure 1. The ω_0 values satisfying the two previous criteria are different for different theories and, in particular, they are smaller when the upper bound on Ω_B are larger (see table). The baryon contribution to the total density parameter cannot be therefore arbitrarily high because, for too large Ω_B values, the resulting cosmological models will not be then compatible with solar system experiments, which require $\omega_0 > 500$. The allowed interval found for Ω_B is

$$0.01 \leq \Omega_B \leq 0.78$$

When a wider observational range for the ${}^7\text{Li/H}$ abundance is considered (${}^7\text{Li/H} \leq 6 \cdot 10^{-10}$, which corresponds to a depletion by a factor of 4), this interval becomes

$$0.01 \leq \Omega_B \leq 0.98$$

In order to illustrate more precisely our procedure, we plot on figures 2, the light elements primordial abundances for GR and for two particular theories appearing on figure 1. These two theories are defined

²Our code solves the scalar-tensor cosmological equations by using a 6th order Runge-Kutta integration, while the primordial nucleosynthesis rate equations are integrated by the Beaudet & Yahil (1977) scheme. The outputs of this code were extensively tested and compared to those obtained by other authors, as well as to the analytical solutions known for some particular cases. It was then used in previous works as, e.g., Serna *et al.* (1992), Serna & Domínguez-Tenreiro (1992, 1993), Serna & Alimi (1996a, 1996b)

by $\lambda^2 = 0.4$, $\log_{10}(\lambda^2/k) = 7.8$ and $\omega_0 = 1.3 \cdot 10^9$ (square symbol), $\lambda^2 = 0.2$, $\log_{10}(\lambda^2/k) = 8$. and $\omega_0 = 5.75 \cdot 10^3$ (circle symbol). On these figures we see clearly that a range of large Ω_B values exists, where predicted primordial abundances are all compatible with observations. Any smaller upper bound on Ω_B than those seen on these figures, can be however obtained by considering larger ω_0 values. We note also that, in fact, the most constraining light elements are ^4He and ^7Li . Any modification on the observational limits on these two elements would imply a different allowed range for Ω_B . In particular, we see from figure 2d, that an almost flat Universe ($\Omega_B \simeq 1$) is permitted when the upper observational limit for $^7\text{Li}/\text{H}$ is $6 \cdot 10^{-10}$.

3. Discussion and Conclusions

A homogeneous and isotropic universe with a baryonic density very close to that needed for closure by baryons is then possible in the framework of class-4 scalar-tensor theories even when the Universe is composed solely by known particles.

Since ω_0 is much higher than the minimum value needed to assure compatibility with all post-Newtonian experiments, these theories are at present very close to GR. However, their behavior at early times is very different from the usual one and can imply a universe expansion rate during the nucleosynthesis process several hundred of times faster than in the FRW cosmology.

The achievement of such ρ_b values can be explained in the following way. The Universe expansion rate at the beginning of primordial nucleosynthesis is slower than in GR. This implies a smaller freezing-out temperature and, hence, a tendency in these theories to underproduce ^4He . The smaller ω_0 , the larger this trend is. The ^4He underproduction can be balanced by considering larger ρ_b values which in principle could imply an excessive D burning. But, contrary to all other attempts of modifying the standard hot big bang model, the Universe expansion rate obtained in class-4 theories becomes during BBN faster than in GR. Consequently, the D burning is not very effective because it occurs in a shortest time and, hence, large ρ_b values are possible.

Obviously, in order to evaluate **quantitatively** the light element production in these theories, it was necessary to consider a specific form for $\omega(\Phi)$ (Eq. 1). However, this form already reveals a large number of models for which Ω_B is much larger than in GR. Furthermore, it is almost evident that such a result should be also obtained for any other scalar-tensor theory, defined by a different coupling function, but

implying a similar non-monotonic behavior for the speed-up factor. As a matter of fact, in order to avoid both an overproduction of ${}^4\text{He}$ and an underproduction of D , the only condition is that the speed-up factor be smaller than unity at the beginning of BBN and becomes, during the nucleosynthesis process, larger than in GR.

It is important to note, that we do not claim here that the Ω_B value is necessarily high. The essential point that we want to stress is that, in a homogeneous and isotropic universe composed only by known particles, primordial nucleosynthesis does not always impose a very stringent bound on the baryon contribution to the density parameter.

Finally, it must be also pointed out that the study of inflation models in the framework of these theories, where a form of the coupling function is now known (Eq. 1), and their consequences on the primordial density fluctuation spectrum, as well as on the formation of large-scale structures are now important open questions which could lead to a new cosmological scenario.

λ^2	$\Omega_B^{max} = 0.3$		$\Omega_B^{max} = 0.4$		$\Omega_B^{max} = 0.5$		$\Omega_B^{max} = 0.6$		$\Omega_B^{max} = 0.7$		$\omega_0^{max} = 500$
	κ	ω_0^{max}	κ	ω_0^{max}	κ	ω_0^{max}	κ	ω_0^{max}	κ	ω_0^{max}	κ
0,20	0,98	1,46e12	1,45	8,80e12	2,11	1,03e13	4,20	2,02e11	6,17	4,00e08	8,714
0,25	1,00	1,50e13	1,56	6,33e13	3,00	1,38e13	6,30	3,45e09	7,30	4,00e07	9,262
0,30	1,02	8,00e13	1,80	2,30e14	5,90	5,20e10	7,30	5,00e08	8,00	1,30e07	9,674
0,35	1,06	3,20e14	2,30	3,80e14	7,10	5,00e09	7,90	2,10e08	8,50	6,50e06	9,997
0,38	1,10	6,00e14	3,00	1,67e14	7,50	2,30e09	8,20	1,10e08	8,70	5,00e06	10,135
0,40	1,14	1,00e15	5,10	2,50e13	7,80	1,30e09	8,40	8,50e07	8,90	3,50e06	10,260
0,45	1,20	2,65e15	6,90	5,00e10	8,20	8,50e08	8,80	4,00e07	9,20	2,70e06	10,478
0,50	1,40	5,00e15	7,60	1,00e10	8,60	3,50e08	9,10	2,50e07	9,50	1,50e06	10,666
0,60	1,80	1,40e16	8,40	4,00e09	9,10	2,50e08	9,50	2,20e07	9,90	1,05e06	10,967
0,70	3,10	3,83e15	8,90	2,00e09	9,50	1,50e08	9,80	1,90e07	10,2	9,00e05	11,204
0,80	7,10	8,00e11	9,20	1,10e09	9,80	8,00e07	10,1	1,25e07	10,5	7,25e05	11,395
1,00	8,50	5,00e10	9,70	1,00e09	10,2	7,00e07	10,5	9,50e06	10,8	6,50e05	11,688

$$\kappa \equiv \log_{10}(\lambda^2/k)$$

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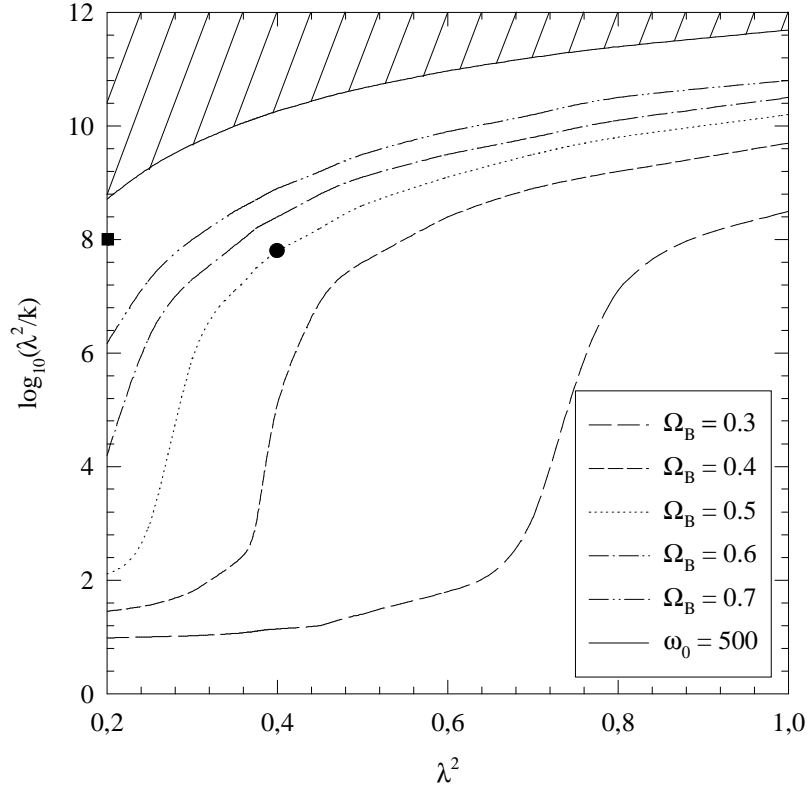


Fig. 1.— Space of theories leading to different upper bounds on Ω_B : The observational constraints used to construct this figure are: $0.221 \leq Y_p \leq 0.243$, $D/H \geq 1.5 \cdot 10^{-5}$, $(D+{}^3\text{He}) \leq 1.1 \cdot 10^{-4}$, $0.7 \cdot 10^{-10} \leq {}^7\text{Li}/H \leq 3.5 \cdot 10^{-10}$, and $h_0 \in [0.4, 1]$. The two symbols (square, circle) correspond to the two particular theories commented in the text and presented in figures 2. The dashed region represents the space of theories with an upper limit on Ω_B requiring an ω_0 value smaller than 500.

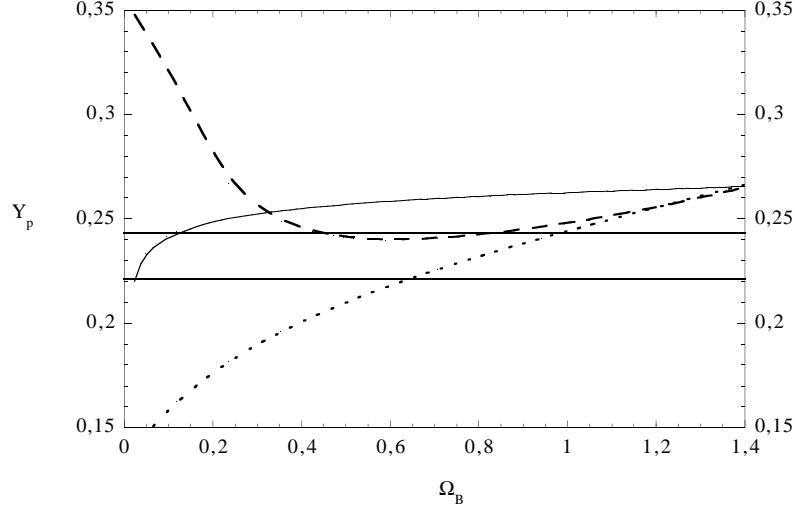


Fig. 2.— Primordial abundances of a) ${}^4\text{He}$, b) D/H , c) $(\text{D}+{}^3\text{He})/\text{H}$, and d) ${}^7\text{Li}$, as a function of Ω_B . The theories shown in this figure are defined by $\lambda^2 = 0.4$, $\log_{10}(\lambda^2/k) = 7.8$, $\omega_0 = 1.3 \cdot 10^9$ (dashed line, corresponding to the circle symbol on figure 1), $\lambda^2 = 0.2$, $\log_{10}(\lambda^2/k) = 8$, $\omega_0 = 15.75 \cdot 10^3$ (dotted line, the square symbol on figure 1) and GR (solid line). A wider observational range for the ${}^7\text{Li}/\text{H}$ abundance (${}^7\text{Li}/\text{H} < 6 \cdot 10^{-10}$ dashed-dotted line) is also displayed

